

Exercise 42

Find the derivative of the function.

$$y = \sqrt{x + \sqrt{x + \sqrt{x}}}$$

Solution

Take the derivative using the chain rule.

$$\begin{aligned} y' &= \frac{dy}{dx} = \frac{d}{dx} \left(\sqrt{x + \sqrt{x + \sqrt{x}}} \right) \\ &= \frac{1}{2} \left(x + \sqrt{x + \sqrt{x}} \right)^{-1/2} \cdot \frac{d}{dx} \left(x + \sqrt{x + \sqrt{x}} \right) \\ &= \frac{1}{2} \left(x + \sqrt{x + \sqrt{x}} \right)^{-1/2} \cdot \left[1 + \frac{1}{2} (x + \sqrt{x})^{-1/2} \cdot \frac{d}{dx} (x + \sqrt{x}) \right] \\ &= \frac{1}{2} \left(x + \sqrt{x + \sqrt{x}} \right)^{-1/2} \cdot \left[1 + \frac{1}{2} (x + \sqrt{x})^{-1/2} \cdot \left(1 + \frac{1}{2} x^{-1/2} \right) \right] \\ &= \frac{1 + \frac{1}{2} (x + \sqrt{x})^{-1/2} \cdot \left(1 + \frac{1}{2} x^{-1/2} \right)}{2\sqrt{x + \sqrt{x + \sqrt{x}}}} \\ &= \frac{1 + \frac{1}{2\sqrt{x + \sqrt{x}}} + \frac{1}{4\sqrt{x}\sqrt{x + \sqrt{x}}}}{2\sqrt{x + \sqrt{x + \sqrt{x}}}} \\ &= \frac{\frac{4\sqrt{x}\sqrt{x + \sqrt{x}}}{4\sqrt{x}\sqrt{x + \sqrt{x}}} + \frac{2\sqrt{x}}{4\sqrt{x}\sqrt{x + \sqrt{x}}} + \frac{1}{4\sqrt{x}\sqrt{x + \sqrt{x}}}}{2\sqrt{x + \sqrt{x + \sqrt{x}}}} \\ &= \frac{4\sqrt{x}\sqrt{x + \sqrt{x}} + 2\sqrt{x} + 1}{4\sqrt{x}\sqrt{x + \sqrt{x}}} \\ &= \frac{4\sqrt{x}\sqrt{x + \sqrt{x}} + 2\sqrt{x} + 1}{8\sqrt{x}\sqrt{x + \sqrt{x}}\sqrt{x + \sqrt{x + \sqrt{x}}}} \end{aligned}$$